

Defect controlled vortex generation in current-carrying narrow superconducting strips

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We experimentally study effect of single circular hole on the critical current I_c of narrow superconducting strip with width W much smaller than Pearl penetration depth Λ . We found nonmonotonous dependence of I_c on the location of a hole across the strip and a weak dependence of I_c on radius of hole has been found in case of hole with $\xi \ll R \ll W$ (ξ is a superconducting coherence length) which is placed in the center of strip. The observed effects are caused by competition of two mechanisms of destruction of superconductivity - the entrance of vortex via edge of the strip and the nucleation of the vortex-antivortex pair near the hole. The mechanisms are clearly distinguishable by difference in dependence of I_c on weak magnetic field.

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I. INTRODUCTION

The maximal current, which can pass through the superconducting strip without dissipation, is equal to $I_{dep} = j_{dep} W d$, where W is a width and d is a thickness of the strip and j_{dep} is the depairing current density. However in experiments, this value can be reached only in relatively thin ($d < \lambda$, λ is the magnetic field penetration depth) and narrow strips with $W < \Lambda = 2\lambda^2/d$ in which a current density is distributed uniformly across the width and thickness [1]. In opposite situation of wide and thick strips ($d > \lambda$, $W \gg \Lambda$), an expelling of magnetic field, which is induced by an applied current, from the interior of the strip leads to piling up of current density at the edge of strip and the maximal critical current becomes smaller than I_{dep} . Moreover, even in the narrow and thin superconducting strips with uniformly distributed current density the measured critical current I_c is smaller than I_{dep} [2–4]. The suppression of experimentally measured critical current in such structures can be explained by a presence of defects in the strip. Under defect we understand a local perturbation of properties of the strip which provides a local suppression (complete or partial) of the superconducting order parameter Δ in the current-carrying state of the strip. This local suppression of Δ can be due to local variation of the thickness/width of the strip, its material parameters such as the electron mean path and/or critical temperature or such suppression of Δ may appear temporally in the place of absorption of high energetic photons or particles. The superconducting current avoids the place with suppressed Δ and this phenomenon leads to nonuniform current distribution, with the maximal current density near the defect place. Now this phenomenon is known as a current crowding effect, which was studied in details theoretically in [5] and confirmed experimentally in [6–9] on model systems

of narrow strips with sharp bends which play a role of defects.

In the recent theoretical work [10], the effect of a single defect, which is located close or far from the edge of the superconducting strip with finite width, on generation of resistive state was studied in frame of Ginzburg-Landau approach. The defect was modelled by the circle with locally suppressed superconductivity - we call it later as a 'hole'. It has been shown that the mechanism of destruction of superconducting state by applied current in a narrow strip depends on position of the hole with respect to the edge of strip. When the hole sits close to the edge, the self-generated vortices enter the hole via the edge of the film and then they move across the superconductor. In this sense it resembles effect of edge defect [11–14]. When the hole is located pretty far from the edges, the vortex-antivortex pair is nucleated inside the hole and a motion of the pair destroys the superconducting state [10, 15]. Domination of the first or the second mechanism of destruction of superconductivity leads to extremely nonmonotonic dependence of critical current I_c on the position of the hole (see Fig. 4 in Ref. [10]).

The interesting effects arise when the hole is located in the center of strip. First of all the theory predicts that in a certain range of the hole's radius $\xi \ll R \ll W$ the critical current is independent of R and equals to about $0.5I_{dep}$ [5, 16]. This counterintuitive result (one could expect that $I_c = I_{dep}(1 - 2R/W)$ in case of uniform current distribution near the hole and $I_c \simeq I_{dep}$ when $R \ll W$) is the direct consequence of the current crowding near the circular hole. Secondly, in this particular situation the small magnetic field B weakly affects the critical current in comparison to a strip without hole. This weakening of the $I_c(B)$ dependence is because the resistive state is determined by the vortex-antivortex nucleation near the hole and not at the edges of a strip where Meissner cur-

rents are maximal. Moreover, due to opposite direction of the generated Meissner currents at the opposite edges of strip, the influence of external magnetic field on I_c of strips is dependent on location of the hole. This effect leads to increase of I_c when the hole is near to one of the edges and to decrease of I_c when the hole is at the opposite edge (see Fig. 9 in [10]).

We stress here the difference between the effect of defects at relatively large magnetic field (when there is dense vortex matter in the strip) and at zero or small magnetic fields $B \lesssim B_s \simeq \Phi_0/2\pi\xi W$ (when there is no field induced vortices) in current-carrying superconductors. In the former case, the vortices, which are created by the externally applied magnetic field, can be effectively pinned by defects and, thereby, this strengthens transport properties of superconductor. Pinning by intrinsic and artificially created defects has been extensively studied in numerous theoretical and experimental works [17–19]. In the case of small or even zero magnetic field, the superconductivity is weakened near the defect due to current crowding. When the applied current exceeds some critical value the vortices are nucleated near this weak place and their motion destroys the superconducting state [10, 15].

Comparison of the theoretical predictions [10] with experimental results, which are obtained in real structures, is complicated. It is very difficult to find a single natural (intrinsic) defect in a film which localizes far enough from a lot of other similar defects and then arrange experiment to study transport properties which are determined by this particular defect and are not smeared out by collective effect of other defects. In case of local suppression of superconductivity caused by absorbed photon, deterministic localization of absorption site is not possible at all due to probabilistic nature of the photon absorption.

In the present work we made holes of different size and placed them in different places across the superconducting strip. Such a model system mimics the main effect of the real defect - current crowding effect and it allows us in controllable manner to study the role of the defects on the critical current and mechanisms of destruction of the superconducting state.

We have experimentally found the non-monotonous dependence of I_c of strips with the hole with constant radius R on the position of this hole with respect to the edge of the strip. We have also shown that the dependence of I_c on weak magnetic fields in case of mechanism of vortex-antivortex nucleation in vicinity of the hole is weaker than the $I_c(B)$ -dependence which is determined by a single-vortex penetration through the edge of superconducting strip in agreement with theoretical predictions [10]. Our results allow not only to clarify the role of defects on suppression of the critical current at zero and low magnetic fields ($B \lesssim B_s$, Meissner state) but they also help to understand the mechanism of photon detection by current-carrying superconducting strips.

II. EXPERIMENT

NbN films with thickness about 5 nm were deposited on a heated sapphire substrate by DC reactive magnetron sputtering of pure Nb target in a gas mixture atmosphere of argon and nitrogen. For the used deposition conditions of the total pressure $2.8 \cdot 10^{-3}$ mbar, partial pressure of nitrogen $3.2 \cdot 10^{-4}$ mbar and sputter current 150 mA, films with the critical temperature $T_c \simeq 13$ K, square resistance of about 300 Ohm and residual resistance ratio $RRR = 0.89$ were deposited.

Patterning of films has been done by electron-beam lithography and ion-milling technique. The films were patterned into single-bridge structures with a width about $1 \mu\text{m}$ and a length $20 \mu\text{m}$ which were embedded between two millimeter sized contact pads. The gradual transition between the bridge and the contacts with radius of curvature $r = 5 \mu\text{m}$ was realized to avoid a current crowding at T-shape connections which were considered in details in [5].

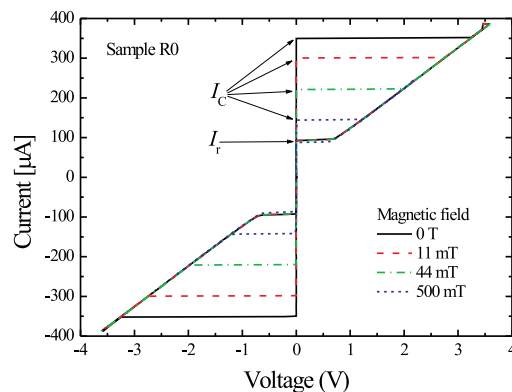


FIG. 1: Current-voltage characteristics of the superconducting strip without hole at different magnetic fields indicated in the legend. The arrows indicate correspondent critical currents I_c and current I_r of recovery of the bridge into superconducting state.

Two series of samples (24 bridges each) have been fabricated on separated 10 mm squared chips. In the first series of samples, circle with radius $R \simeq 130$ nm was etched inside a bridge. The etched circles were always placed at a half-length of bridge. The coordinate of circles across the width of bridge was varied with respect to the left edge of bridge with a step $\delta x \simeq 35$ nm between two subsequent positions (see the insets in Fig. 2). Through the paper samples which belong to this series of bridges will be named XNN where NN is a distance in nanometer between left edge of a strip and center of a hole which is etched inside the strip. The radius of circles with a center placed at the edge of bridges (in this case it is better to talk about semi-circles, see the top left inset in Fig. 2) was by a factor $\sqrt{2}$ larger i.e. about 185 nm.

This was done to keep constant the area with suppressed order parameter independently of location of the hole.

In the second series of samples, the position of etched circle (the hole) was kept constant in the middle (half-length, half-width) of a strip but the radius of hole was varied by an order of magnitude from about 29 nm up to 225 nm. In the paper, samples, which belong to this series, will be named RNN where NN is a radius of etched hole in nanometer. Real dimensions of fabricated structures (width of bridge, W , radius of hole, R , and its position with respect to the left edge of bridge, coordinate x) were measured after patterning by a scanning electron microscopy (SEM).

All bridges were characterized by their critical temperature of superconducting transition. The T_c value was determined as a temperature at which resistance of measured structure drops below 0.1% of the normal state resistance measured at temperatures approximately twice higher than T_c . The obtained spread of T_c among bridges in each series was about 0.3 K which can be attributed to slight variation of superconducting strength over the film caused by, for example, non-uniformity of thickness of deposited film. In case of ultra-thin films with thickness about the coherence length which is in case of NbN is about 4-5 nm [20], even a small difference in thickness leads to relatively large change in T_c [21, 22].

The current-voltage characteristics (IV-curves) were measured in a cryogenic-free system at $T = 5.1$ K in a DC current-bias mode. The magnetic field up to 1 T generating by a superconducting solenoid was applied normally to a sample surface. From the previous studies [20] we know that the field, which is required for penetration of magnetic vortex into a micrometer wide bridge made from thin NbN film, is in the range of few tens millitesla and therefore the available in our experiments range of B is enough for detailed investigations of bridges in Meissner and in the vortex states. The minimum step of variation of magnetic field was 1 mT which is determined by accuracy and stability of a used power supply of superconducting solenoid.

Typical IV-curves for four different magnetic fields are shown in Fig.1. The curves were hysteretic independently of the magnetic fields in the whole range of applied fields B . At a critical current I_c , the bridge is characterized by a sharp jump from superconducting to normal state. Recovery of superconductivity in the bridge happens at current I_r which is almost independent of magnetic field as it is clearly seen in Fig. 1. Contrary, I_c demonstrates significant decrease with increase of the applied field. The accuracy of I_c determination in our experimental setup is 0.5 % of I_c , i.e. about 1 μ A for studied range of critical currents.

III. RESULTS

The experimentally obtained dependence of the critical current I_c on position of the hole is shown in Fig. 2.

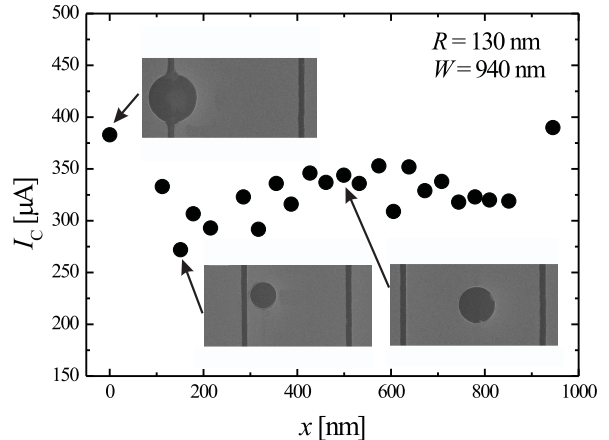


FIG. 2: Experimental dependence of the critical current on the position of the hole in the strip (series "X"). Width of the strip is 940 nm, radius of the hole is 130 nm $\sim W/7$ ($R=185$ nm when semi-hole is placed at the edge). In the insets we present SEM images of the strips with the holes in different positions.

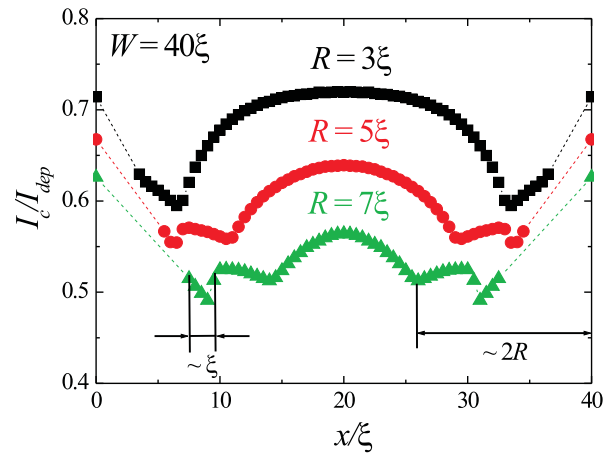


FIG. 3: Theoretical dependence of the critical current on coordinate of hole's center in the strip with width $W = 40\xi$. Results are shown for holes with three radii.

It is seen that I_c is maximal when the hole is in the middle of the strip and when the hole (semi-circle) is placed at the edge of the strip. Shift of position of the hole from the center of the strip leads to gradual decrease of I_c which reaches the minimum values when the hole is close to the edge. This experimental result qualitatively coincides with the prediction of Ref. [10]. In Fig. 3 we present results of theoretical calculations which were performed in the framework of Ginzburg-Landau model for

different holes and strip with $W = 40\xi$ (details of numerical calculations are present in Ref. [10]). In contrast to Ref. [10] we determine the critical current as the current at which permanent vortex flow starts in the strip with a hole. In Ref. [10] I_c was defined as the current at which the first vortex enters the hole when it is located near the edge of the strip - due to assumption that the 'hole' is created by the absorbed photon and it should dissolve after some finite time interval. New definition leads only to quantitative difference with result of Ref. [10]: i) the local minimum in dependence $I_c(x)$ now reaches when the hole sits on the finite distance from the edge; ii) additional minima appear when the hole practically touches the edge. These minima appear for relatively wide strips $W \gtrsim 30\xi$ and large radii of the holes $R \gtrsim 3\xi$. The width of the additional minima is about the coherence length (see Fig. 3) and it is rather difficult to resolve it in the experiment due to small $\xi \simeq 5$ nm in NbN. Small width of this minima ($\sim \xi$) points on its origin - it appears due to suppression of the order parameter in the narrow sidewalk between edge of the hole and edge of the strip when its width becomes about ξ (because we model the hole as a region with locally suppressed superconductivity). It results in decrease of the 'superconducting' width of the strip.

We also studied how I_c depends on the radius of the hole when it is placed in the center of the strip. Theoretical results are present in Fig. 4 while experimental $I_c(R)$ -dependence is shown in Fig. 5. The theory predicts, that when $\xi \ll R \ll W$ the critical current very weakly depends on R (see inset in Fig. 4). Calculations in the London model [5, 16] predict that in this case $I_c/I_{dep} = 1/2$ while our numerical results give a little larger value $I_c/I_{dep} \simeq 0.65 - 0.7$. The difference between theoretical results is most probably connected with the characteristics of the hole. In Refs. [5, 16] the hole was considered as a 'well' with jump of Δ at the edge while in our model, due to proximity effect, Δ changes from zero to its maximal value on the distance $\sim \xi$ near the hole's edge.

In the experiment we find small variation of $I_c \simeq 284 - 335 \mu\text{A}$ ($I_c/I_{dep} \simeq 0.42 - 0.49$) for holes with radius $R \simeq 26 - 76$ nm ($R \simeq 5 - 15\xi$) - see Fig. 5. The depairing current $I_{dep} = 679 \mu\text{A}$ has been calculated in frame of Ginzburg-Landau theory with account of the temperature dependent correction factor which was obtained by Kupriyanov and Lukichev [23] for dirty limit superconductors (see Eqs. 3 and 4 in [20]). We have to note, that even for strips without hole $I_c/I_{dep} \simeq 0.5$ (see Fig. 5) due to presence of the intrinsic defects. These defects may be present also at the edge of the hole which additionally suppresses the critical current and it could be a reason for dispersion of I_c for holes with radius $R = 26 - 76$ nm and smaller value of I_c than expected from the theory.

Theory [10] predicts, that when the hole is placed in the center of the strip the resistive state starts via nucleation of the vortex-antivortex pair near the hole. Direct experimental visualization of this process is practically

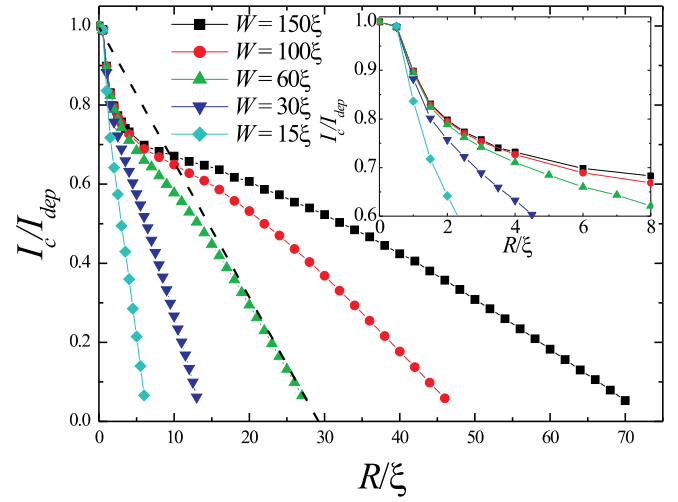


FIG. 4: Calculated dependence of I_c on radius of hole placed in the center of the superconducting strip with different width indicated in the legend. Dashed line corresponds to linear dependence $I_c/I_{dep} = 1 - 2R/W$ for the strip with $W = 60\xi$. In the inset we zoom the region of small radii $R \ll W$.

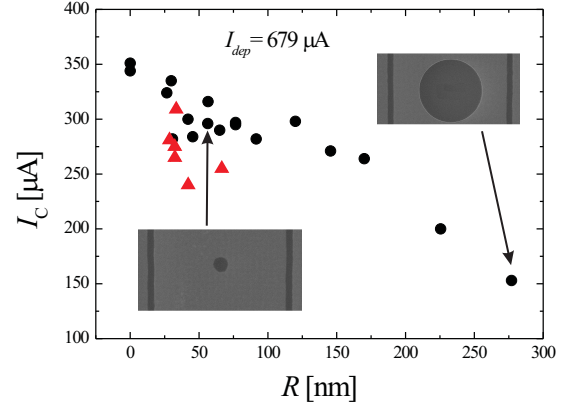


FIG. 5: The experimentally measured critical current of strips ($W = 980$ nm) with hole of different radius, which is placed in the center of the superconductor (series "R"). In the inset we show SEM images of a center part of strip with hole. Triangles mark the samples with linear $I_c(B)$ dependence at low B which indicates that vortices in these samples are generated at the edges.

impossible at the present time, due to very short time scale (\sim picoseconds) of vortex nucleation. However it could be checked indirectly, using external magnetic field B . The applied magnetic field induces screening current which is maximal near the edges and it is equal to zero in the center of the strip. Therefore if resistive state is realized via vortex entrance via edge of the film then applied magnetic field strongly influences I_c . Contrary, if resistive state is realized via generation of the vortex-antivortex pairs near the central part of the strip

the same magnetic field weakly affects I_c . Experimental $I_c(B)$ -dependencies will differ significantly in dependence on location of penetration of vortex or vortex-antivortex pair and, thereby, can be used for determination of position of vortex generation in the strip at $I > I_c$.

In Fig. 6 we show experimental dependencies $I_c(B)$ for strips with and without hole in the center. For samples R26, R91 and R170 ($R = 26, 91, 170$ nm) there is plateau at low B while for strip without hole (sample R0) I_c decays almost linearly at low B . Therefore we conclude, that in the sample without the hole the vortices are generated at the edge of the strip while in the samples with the hole the vortex-antivortex pairs are generated near the hole at $|B| \lesssim 5\text{--}18$ mT (the larger the hole the larger the threshold magnetic field). Magnetic fields larger the threshold produce large screening currents at the edges of the strip which, together with transport current, exceeds the current density near the hole and the vortices start to be generated at the edge. As a result the strong field dependence of I_c is recovered.

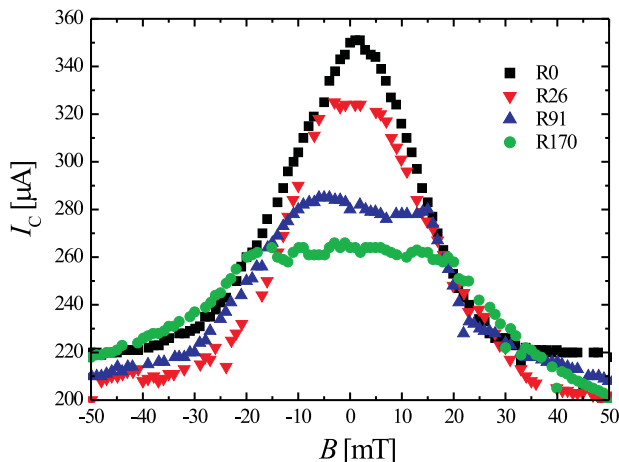


FIG. 6: The dependence of I_c on magnetic field B for strips with hole of radii $R = 26, 91, 170$ nm (R26, R91, R170) in the center of the superconductor and strip without hole (R0).

When the hole is located close to the edge of the strip the maximum of $I_c(B)$ -dependence is shifted to the finite magnetic field (sample X150, black squares in Fig. 7). Nearly linear $I_c(B)$ -dependence at low B again demonstrates that vortices enter via edges and the shift in $I_c(B)$ is connected with different vortex entry conditions from the left and right edges due to presence of the hole. Note, that such a shift was also observed for strip with central hole (sample R32, red circles in Fig. 7) where it most probably appears due to relatively large intrinsic defect at the left edge of the strip (the location of the defect follows from the direction of the shift in $I_c(B)$).

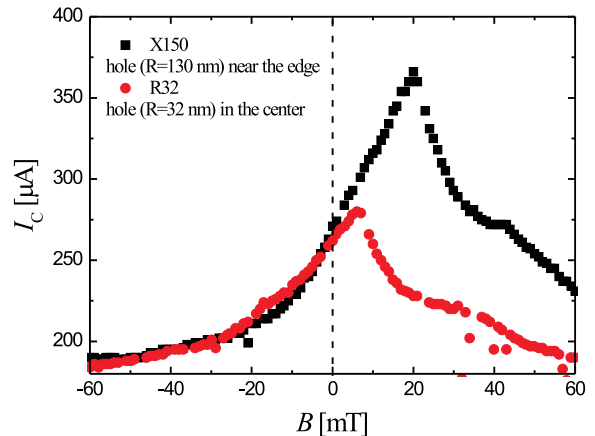


FIG. 7: Dependence of critical current I_c on magnetic field of sample X150 (black squares, hole with $R = 130$ nm is located at the distance $x = 150$ nm from left edge; lower left SEM image in Fig. 2) and sample R32 (red circles; $R = 32$ nm; the hole is in the center of strip).

IV. DISCUSSION

There are three main experimental observations which confirm theoretical predictions formulated in Ref. [10] and [16]:

- i) the non-monotonic dependence of I_c on coordinate x of hole across the width of superconducting strip (Fig. 2);
- ii) the dependence of I_c on radius of hole which is placed in the middle of the strip is much weaker than it is expected from geometrical point of view when radius of hole is in the range $\xi \ll R \ll W$ (Fig. 5);
- iii) the weak dependence of I_c on magnetic field at small B when the hole is located in the middle of strip (Fig. 6). This weak $I_c(B)$ -dependence is caused by generation of vortex-antivortex (V-A) pairs near the hole due to a current crowding in vicinity of that place. This area is weakly influenced by Meissner currents which are mainly concentrated near edges of strip. Contrary, the strong, linear decrease of I_c with increasing B is observed when vortex is generated at the edge of strip. This mechanism dominates over V-A-generation mechanism mostly in strips with holes which are shifted from the center of strip or in strips without any artificial holes (Figs. 6 and 7).

Quantitative deviation from the theory we explain by the presence of the intrinsic defects in real strips. Indeed, some strips without hole has a critical current comparable to I_c of the strip with the hole (see Fig. 5). Because measured suppression of I_c by the circular hole with radius $\xi \ll R \ll W$ is not large (see Fig. 6) one can imagine situation when intrinsic defect suppresses I_c stronger than circular hole. This result is supported by the measurement of $I_c(B)$ -dependence which demonstrates linear

variation of I_c at low magnetic field despite the presence of the hole in the center (for illustration see Fig. 7, sample R32). We believe, that asymmetry of $I_c(B)$, visible in Fig. 7 for strip with the central hole also comes from the intrinsic defects near the hole or deviation from the circular shape which also breaks mirror symmetry in the superconducting strip.

Although we made our measurements for relatively wide micron size strip we believe that obtained results and their analysis are at least qualitatively valid for narrower strips too. Our numerical calculations show that the shape of $I_c(x)$ -dependence does not change substantially while one keeps the ratio R/W the same and when $R \gg \xi$. Our theoretical investigation shows that in the strips with width $W = 20 - 60\xi$ the minima of dependence $I_c(x)$ correspond to holes placed on the distance $\sim 2R$ from the edge of the strip which is close to the experimental result for strip with $W \simeq 200\xi$ (see Fig. 2).

From Fig. 3 it is also seen that I_c is almost independent of the coordinate in a pretty wide range of x in case the smallest hole ($R = 3\xi$) which is placed in the vicinity of center of the strip. The plateau in $I_c(x)$ -dependence decreases with increase of R and, in case of the largest radius, a sharp maximum of I_c is seen in the middle of strip.

Note, that when radius of the hole $R \ll W$ and $R \lesssim 4\xi$ there is relatively strong dependence of I_c on R (see inset in Fig. 4). It results in smaller value of I_c for the strip with hole at the edge (in the form of semicircle

with radius $\sqrt{2}R$) than for the strip with central hole (it could be seen in Fig. 3). For larger radii we have opposite situation (see Fig. 3 and Fig. 2).

Our results could be applied for understanding of some properties of superconducting nanowire single-photon detectors (SNSPD). If one supposes that the absorbed photon creates finite region with suppressed superconductivity and size smaller then W then it will influence the critical current of the nanowire in a similar way like holes studied in this work. Our result directly demonstrates that the resistive state will appear at different currents, depending on the place in the strip where the photon was absorbed. Therefore detection efficiency of SNSPD should change monotonically with variation of the current, the effect which was observed in all experiments on SNSPD (see for example recent review [24]).

Acknowledgments

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